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DECOHERENCE AND VACUUM FLUCTUATIONS

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Abstract

The interference pattern of coherent electrons is effected by coupling to the quantized electromagnetic field. The amplitudes of the interference maxima are changed by a factor which depends upon a double line integral of the photon two-point function around the closed path of the electrons. The interference pattern is sensitive to shifts in the vacuum fluctuations in regions from which the electrons are excluded. Thus this effect combines aspects of both the Casimir and the Aharonov-Bohm effects. The coupling to the quantized electromagnetic field tends to decrease the amplitude of the interference oscillations, and hence is a form of decoherence. The contributions due to photon emission and to vacuum fluctuations may be separately identified. It is to be expected that photon emission leads to decoherence, as it can reveal which path an electron takes. It is less obvious that vacuum fluctuations also can cause decoherence. What is directly observable is a shift in the fluctuations due, for example, to the presence of a conducting plate. In the case of electrons moving parallel to conducting boundaries, the dominant decohering influence is that of the vacuum fluctuations. The shift in the interference amplitudes can be of the order of a few percent, so experimental verification of this effect may be possible. The possibility of using this effect to probe the interior of matter, e.g., to determine the electrical conductivity of a rod by means of electrons encircling it is discussed. The effect of squeezed states of the photon field are considered, and it is shown that such states may either enhance or suppress the decohering effects of the vacuum fluctuations.

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Vacuum Fluctuations and Photon Emission

It is well known that the interaction of a quantum system with its environment can destroy quantum coherence. In this paper, a particular example of this phenomenon will be discussed: the coupling of coherent electrons to the quantized radiation field. This coupling gives rise both to the possibility of photon emission and of interaction of the electrons with the electromagnetic vacuum fluctuations. Consider an electron interference experiment in which coherent electrons may travel from x_i to x_f along either of two classical paths, C_1 or C_2. First let us recall the analysis of this experiment when the effects of the electromagnetic field are ignored. Let ψ_1 and ψ_2 be the amplitudes for an electron to travel along C_1 and C_2, respectively. Then the superposed amplitude is $\psi = \psi_1 + \psi_2$, and the number density of electrons detected at x_f is

$$n_0(x_f) = |\psi|^2 = |\psi_1|^2 + |\psi_2|^2 + 2Re(\psi_1 {\psi_2}^*)\,, \tag{1}$$

the last term being responsible for the interference pattern. Note that C_1 and C_2 are *spacetime* paths, as the events of emission and detection of the electrons occur at different times as well as different points in space.

We now wish to couple the electrons to the quantized electromagnetic field and examine the effect upon the interference pattern. This problem has been analyzed in detail in Ref. [1]. Here we will quote and discuss some of the main results. When both photon emission and vacuum fluctuation effects are present, the number density of electrons detected at x_f becomes

$$n(x_f) = |\psi_1|^2 + |\psi_2|^2 + 2e^W Re(e^{i\phi}\psi_1 {\psi_2}^*)\,. \tag{2}$$

Here ϕ is a phase shift introduced by the interaction, and W is a function which describes the change in the amplitude of the interference oscillations (the contrast). The phase shift ϕ includes the Aharonov-Bohm shift due to any classical electromagnetic fields generated by the electrons. However, in this paper we will be primarily concerned with the amplitude of the interference oscillations, as this quantity carries the information about the vacuum fluctuations. The explicit form for W is

$$W = -2\pi\alpha \oint_C dx_\mu \oint_C dx'_\nu D^{\mu\nu}(x,x'), \tag{3}$$

where α is the fine structure constant, and $C = C_1 - C_2$ is the closed spacetime path obtained by traversing C_1 in the forward direction and C_2 in the backward direction. Note that Eq. (3) is independent of the direction in which the integrals around C are taken. The photon Hadamard (anticommutator) function, $D^{\mu\nu}(x,x')$, is defined by

$$D^{\mu\nu}(x,x') = \frac{1}{2}\langle 0|\{A^\mu(x), A^\nu(x')\}|0\rangle\,. \tag{4}$$

By means of the four dimensional Stokes theorem, we may write

$$W = -2\pi\alpha \int da_{\mu\nu} \int da'_{\rho\sigma} D^{\mu\nu;\rho\sigma}(x,x'), \tag{5}$$

where $da_{\mu\nu}$ is the area element of the timelike two-surface enclosed by C, and

$$D^{\mu\nu;\rho\sigma}(x,x') = \frac{1}{2}\langle 0|\{F^{\mu\nu}(x), F^{\rho\sigma}(x')\}|0\rangle \tag{6}$$

is the Hadamard function for the field strengths. Equation (5) has the remarkable interpretation that the electrons are sensitive to vacuum fluctuations in regions from which they are excluded. This is analogous to the situation in the Aharonov-Bohm effect[2], where the phase shift can depend upon classical electromagnetic fields in regions which the electrons cannot penetrate. The spacetime geometry of the paths C_1 and C_2 encircling a region is illustrated in Fig. 1.

In Eq. (2), it was assumed that both photon emission and vacuum fluctuation effects are present at the same time. This is the usual case in an interference experiment in which no attempt is made to detect the emitted photons. However, in principle, it is possible to distinguish the two by means of a *veto* experiment. Suppose that we arrange for the flux of electrons to be sufficiently low that only one electron is in the apparatus at any one time, and that any photons emitted be detected. Whenever a photon is in fact detected, the electron counters are switched off for a sufficient time to insure that the associated electron is not counted. In this way, we guarantee that the interference pattern is comprised only of those electrons which have not emitted photons. In this case, the relevant contrast factor is no longer e^W, but rather e^{W_F}, where

$$W_F = -2\pi\alpha\Big(\int_{C_1} dx_\mu \int_{C_1} dx'_\nu + \int_{C_2} dx_\mu \int_{C_2} dx'_\nu\Big) D^{\mu\nu}(x,x')\,. \tag{7}$$

This function describes the effect of the vacuum fluctuations. Similarly, the effects of photon emission are described by the function

$$W_\gamma = W - W_F = 2\pi\alpha\Big(\int_{C_1} dx_\mu \int_{C_2} dx'_\nu + \int_{C_2} dx_\mu \int_{C_1} dx'_\nu\Big) D^{\mu\nu}(x,x')\,. \tag{8}$$

Note that the vacuum fluctuation effect involves a double line integral over each path separately, whereas the photon emission contribution is a cross term involving a line integral over each path.

Effect of a Conducting Plate

The line integrals in Eqs. (3) and (7) are divergent due to the short distance singularity of $D^{\mu\nu}(x,x')$ when $x \to x'$. This is one of the familiar ultraviolet divergences in quantum field theory. A simple way to avoid dealing with this divergence is to consider only changes in W or W_F due to an external influence, such as the presence of a conducting boundary. In this case, we replace $D^{\mu\nu}(x,x')$ by $D_R^{\mu\nu}(x,x')$, the renormalized Hadamard function obtained by subtracting the empty space function. Let W_R be the function obtained by making this replacement in Eq. (3). As compared to the empty space case, the amplitude of the interference oscillations is multiplied by the factor e^{W_R}. The interference pattern can now become sensitive to

shifts in the vacuum fluctuations, including shifts occurring in excluded regions. In this case, one has an effect which combines aspects of both the Aharonov-Bohm and the Casimir effects[3]. The simplest geometry in which the effects of vacuum fluctuations may be investigated is where one of the electron paths skims above a perfectly conducting plate. Suppose that path C_1 travels for a distance L at a height z above the conducting plate, and later recombines with path C_2, which travels far away from any conductors. Let v be the speed of the electrons. In the limit that $Lc/v \gg z$, i.e. the electron's flight time over the plate is long compared to the light travel time to the plate, we have

$$W_R \approx -\frac{\alpha}{\pi}\Big[1 + log\Big(\frac{Lc}{2vz}\Big)\Big] \,. \tag{9}$$

Note that this is the small z approximation to a function which vanishes in the limit that $z \rightarrow \infty$, as required by the fact that $W_R = 0$ in empty space. We first observe that

$$W_R < 0 \,. \tag{10}$$

This means that the shift in the vacuum fluctuations due to the plate causes a decrease in the contrast, and hence there has been a loss of quantum coherence.

That photon emission can cause decoherence is no surprize. The detection of sufficiently short wavelength photons could reveal which path the electron has taken. It is perhaps less obvious that vacuum fluctuations are also capable of causing decoherence. One might intuitively think of the electrons as being subjected to random force fluctuations which eventually lead to decoherence. Although in the above example $W_R < 0$, there is no reason in principle why one could not have $W_R > 0$, in which case the presence of the conducting boundary would suppress the decohering effects of the vacuum fluctuations.

Squeezed States of the Radiation Field

Such a situation may be displayed explicitly when one replaces the boundary by photons in a squeezed vacuum state. Let the photon field be in squeezed vacuum state for a single mode, which can be defined as[4]

$$|\zeta\rangle = S(\zeta)\,|0\rangle = \exp[\frac{1}{2}\zeta^* a^2 - \frac{1}{2}\zeta (a^\dagger)^2]\,|0\rangle \,, \tag{11}$$

where $\zeta = re^{i\delta}$ is an arbitrary complex number. It may be shown that, in this state,

$$\langle a^\dagger a\rangle = \sinh^2 r \,, \tag{12}$$

and

$$\langle a^2\rangle = -e^{i\delta}\sinh r \cosh r \,. \tag{13}$$

We wish to calculate the shift in W from the actual vacuum state. This shift, W_R, is given by Eqs. (3) and (4) with the expectation value taken in the state $|\zeta\rangle$. The

photon operator product is now understood to be normal ordered with respect to the vacuum state. The result is

$$W_R = -4\pi\alpha|\eta|^2 \sinh r\,[\sinh r + \sin(2\theta+\delta)\cosh r]\,, \tag{14}$$

where

$$\eta = |\eta| e^{i\theta} = \oint_C dx_\mu f^\mu(x)\,, \tag{15}$$

and $f^\mu(x)$ is the mode function of the excited mode. The key point is that one can arrange for W_R to have either sign by an appropriate choice of the state parameter δ. Thus the effect of the squeezed state can be either to enhance or suppress decoherence. In the above example, both vacuum fluctuation and photon emission contributions were included. However, one can treat them separately and show that the shifts in each of W_V and W_γ can have either sign. That squeezed states can suppress the effects of quantum fluctuations below the vacuum level is well known. A squeezed vacuum state necessarily has the expectation value of the energy density negative in certain regions as a result of this suppression. A phenomenon somewhat analogous to the present situation arises when one considers the coupling of a spin system in a classical magnetic field to the electromagnetic vacuum fluctuations. These fluctuations tend to cause a depolarization of the system. Photons in a squeezed state can temporarily reduce this effect, and cause the mean magnetic moment of the system to *increase* relative to the vacuum value[5].

Probing the Interior of Matter

Let us now return to the issue of nonlocality, the ability of electrons to serve as remote probes. A simple illustration of this would arise in the following experiment: send electrons around either side of a cylinder of radius R filled with a material of finite electrical conductivity, as illustrated in Fig. 2. Also arrange that the outer wall of the cylinder is made of a material of very high conductivity. This wall both excludes the electrons and insures that the renormalized field strength Hadamard function, $D_R^{\mu\nu;\rho\sigma}(x,x')$, outside the cylinder is independent of the material on the interior. However, the change in the interference pattern contrast due to the presence of the cylinder also depends upon $D_R^{\mu\nu;\rho\sigma}(x,x')$ inside the cylinder, and hence upon the conductivity of the material on the interior. An explicit calculation of $D_R^{\mu\nu;\rho\sigma}(x,x')$ in the interior requires a knowledge of the dielectric function $\epsilon(\omega)$ of the metal in question. However, we may make an order of magnitude estimate without this detailed information. We may adapt Eq. (9) to obtain an estimate of W_R for the present situation. Let λ_P be the wavelength associated with the plasma frequency in the metal in question. It is essentially the cutoff wavelength, in that only modes whose wavelengths are of this order or longer are effected by the interior material. It plays a role analogous to z in Eq. (9). Our estimate will be based upon the assumption that a result of the form of Eq. (9) also holds in the present situation. If we set

$z \approx \lambda_P$ and $L \approx R$, the radius of the rod, our estimate for W_R may be written as

$$W_R \approx -10^{-3} ln\Big(\frac{cR}{v\lambda_P}\Big), \qquad R \gg \lambda_P, \qquad v \ll c. \tag{16}$$

The result is only weakly (logarithmically) dependent upon the cutoff, λ_P. For example, if $R = 1cm$, $\lambda_P = 810\mathring{A}$ (the approximate plasma wavelength of aluminum), and $v = 0.1c$ (corresponding to $2.5\, keV$ electrons), we find that $W_R \approx -10^{-2}$. Because the amplitude of the interference oscillations is proportional to $e^{W_R} \approx 1 + W_R$, we see that the effect of the presence of the rod is to decrease this amplitude by about 1% in this example. If one were to use a material with a different plasma wavelength, λ'_P, the amplitude shift will now be $W'_R = W_R + \Delta W_R$, where

$$\Delta W_R \approx 10^{-3} ln(\lambda'_P/\lambda_P). \tag{17}$$

This change will typically be of the order of a few times 10^{-4} of the interference oscillation amplitude. Thus measuring the electron interference pattern to this accuracy would enable one to ascertain whether the material in the rod is, for example, aluminum or magnesium ($\lambda'_P = 1170\mathring{A}$). The effect which we have estimated is primarily due to the effects of the vacuum fluctuations, rather than photon emission, as the magnitudes of the integrals in Eq. (8) are determined by the separation of the paths, which is at least as large as R.

Experimental Prospects

Let us conclude with some remarks on the possibility of experimentally verifying the effects discussed in this paper. The effect discussed in the previous paragraph seems to be large enough to be possibly measureable. However, in this case, the theoretical prediction, Eq. (16), is only an order of magnitude estimate. For the case of electrons moving parallel to a conducting plate, the theory is more clear cut. In principle, W_R as given by Eq. (9) could be made arbitrarily large if one could make L large and z and v small. In an actual experiment, the image charge force experienced by an electron will limit the possible range of parameters. In particular, the image effect is negligible so long as

$$L \ll (6cm)\Big(\frac{v}{c}\Big)\Big(\frac{z}{1\mu}\Big)^{\frac{3}{2}}. \tag{18}$$

It is not difficult to satisfy Eq. (18) with choices of parameters that yield a contrast shift, $|W_R|$, of the order of 1%. For example, if we again let $v = 0.1c$ and take $L = 1\,m$ and $z = 1\,mm$, then Eq. (18) is satisfied, and we have $|W_R| \approx 2.2\%$. Thus detection of the vacuum decoherence effect seems to be a possibility.

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Figure Captions

- [1] Electrons traverse spacetime paths C_1 and C_2 to reach point $(\mathbf{x}, t)$. The interference pattern depends upon vacuum fluctuations in the crosshatched region in the interior of these paths, which is the domain of integration in Eq. 5. The cylinder is the world history of an object contained within the electron paths. The spatial projections of the spacetime paths C_1 and C_2 onto the xy plane are C_1' and C_2', respectively. The shaded region in this plane is the cross section in the xy plane of the object around which the electrons travel.

- [2] A "coaxial cable" consisting of an outer cylinder which prevents electrons from penetrating to the interior, and an inner rod, with radius R, of material of finite conductivity.